C.P. No. 85
(13,863)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
CURRENT PAPERS

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1952

Price 3s 6d net

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A Brief Survey of Some Methods and Information Concerning the Aerodynamic Derivatives of Wings in Unsteady Motion at Transonic and Supersonic Speeds

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14th March, 1951

§1. Introductory Remarks

A considerable amount of theoretical work has been done on the problem of the oscillating wing in supersonic flow, but, unfortunately, most theories are restricted to low values of the frequency parameter and to rigid wings describing simple pitching and vertical translational oscillations. From the flutter point of view much more information is required. Theories valid to first order only in the frequency are likely to be inadequate except possibly for guided weapons. Since also wing bending and twisting must be taken into account present theory is of little use for wings with subsonic leading edges (Fig. 1A). In the case of purely supersonic flow, such as that shown in Fig. 1B, the derivatives corresponding to any general mode of distortion can be calculated exactly (subject to the limitations of linearized theory) as in Refs. 5 and 6, while for wings of the type shown in Fig. 1C the purely supersonic region (Region I) can be dealt with as type B, but the mixed supersonic region (Region II) is more difficult to treat except for simple plan forms and modes of oscillation. See Watkins[16,17] and Acum[8].

In most of the theories the assumption is made that the wing is of small thickness but measurements made at the National Physical Laboratory of the pitching moment of a two-dimensional 7½%-thick biconvex aerofoil oscillating at supersonic speeds (Ref. 39) and the theoretical work of Jones[4,28] suggest that thickness has a considerable effect even for 5%-thick wings. It remains to be seen whether such effects will also be important in three dimensions. As yet no measurements of derivatives on wings of finite aspect ratio have been made.

As far as numerical theoretical derivatives are concerned several sets of tables exist for the two-dimensional case (see §2 below) but the only tables covering the flutter range of frequency parameter for a three-dimensional wing appear to be those given by Acum[8] (Table 1 of the report) and Ting-Yi Li[51] for the rectangular wing.

Some of the earlier papers on the subject, not referred to later, are due to Possio[45], Hönl[46], Schwarz[47], Borbely[48], Todd[49] and Collar[50].

§2/

§2. Two-dimensional Theory and Experiment

The linearized theory of thin two-dimensional wings oscillating in a supersonic stream without change of shape except possibly for movement of ailerons has been considered by Temple and John[1], Garrick and Rubinow[2], and others, and may be considered as fairly complete. Other motions, e.g., those of a distorting two-dimensional wing, could be treated by this method without much difficulty. Graphs and tables of derivatives are given in Refs. 1 and 2 and also in Ref. 43. The most significant result of the theory appears to be that negative aerodynamic damping in pitch is possible for certain combinations of Mach number, frequency and axis position (W. P. Jones[3]).

The problem of two-dimensional wings of non-zero thickness has been considered by Jones and Skan[4] for a 5%-thick biconvex aerofoil, and this indicates that thickness may have a considerable effect on the derivatives. See also Jones[28].

The experimental work of Bratt and Chinneck[39] on the pitching of two-dimensional aerofoils at supersonic speeds showed no agreement with thin flat plate theory, the negative aerodynamic damping predicted at certain supersonic speeds not being observed, and only qualitative agreement with Jones' second-order theory (see Refs. 28 and 40).

Sewell[44] has shown that the assumption made in the theory mentioned above[1], that the shock line from the leading edge is straight, does not affect the result.

Carrier[26] has treated the case of an infinite wedge oscillating in a two-dimensional stream.

§3. Methods Used in Dealing with Three-dimensional Supersonic Oscillating Wings

(i) The linearized equation of isentropic, irrotational supersonic flow, is

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)^2 \phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}, \qquad \ldots(1)$$

the motion of the air stream being in the x-direction. (c = speed of sound, v = speed of stream.)

An elementary solution (supersonic source) of this equation, is

$$\phi_0 = \frac{A(\xi,\eta,\zeta)}{r}\left[f\left(t - \frac{M}{c}\frac{x-\xi}{M^2-1} - \frac{r}{c}\right) + f\left(t - \frac{M}{c}\frac{x-\xi}{M^2-1}\frac{x-\xi}{M^2-1} + \frac{r}{c}\right)\right] \qquad \ldots(2)$$

where $$r = \frac{1}{M^2-1}\left\{(x-\xi)^2 - (M^2-1)\left[(y-\eta)^2 + (x-\zeta)^2\right]\right\}^{\frac{1}{2}}$$

and f is an arbitrary function. (M = Mach number = v/c.)

One/

One method of approaching the problem is to assume that the flow round the wing may be regarded as being caused by a distribution of such sources over the plane of the wing, assumed to be the plane $z = 0$, and express the boundary conditions in terms of this distribution.

This leads in general to an integral equation for ϕ, though for regions of purely supersonic flow (that is, regions where the flows over the upper and lower surfaces are independent) the problem reduces to one of evaluating a definite integral for ϕ. See Garrick and Rubinow[5]. Thus, though the evaluation of the velocity potential in purely supersonic regions may be performed, this approach does not lead in general to a solution for the wing as a whole owing to the difficulty of solving the integral equation for mixed supersonic regions, i.e., regions where both the upper and lower surfaces affect each other.

(ii) Starting from the linearized differential equation and using a form of Green's theorem, W. P. Jones[6] has shown that it is possible to construct integrals and integral equations for the velocity potential, for purely supersonic and mixed supersonic regions respectively. Thus, in general, the same difficulty arises in this method as in (i). This method assumes that the motion is simple harmonic.

(iii) In particular cases it may be possible to construct solutions of the linearized differential equation by separating the variables (e.g., Robinson[7]), but even in simple cases such as the delta wing the numerical evaluation of the derivatives would probably be lengthy. It does not seem likely that this method could be applied to any but comparatively simple plan forms.

(iv) It is sometimes possible to show that the velocity potential may be reduced to the sum of a number of conical flows, i.e., flows with velocity potential of the form $\phi = x^n f\left(\frac{y}{x}, \frac{z}{x}\right)$ and evaluate these using the fact that the problem then reduces to the solution of second-order linear partial differential equations with appropriate boundary conditions. This can sometimes be done numerically (Acum[8]), but this approach suffers from the defect that it applies only to simple plan forms and is rather laborious.

(v) The difficulty mentioned in connection with methods (i) and (ii) can sometimes be overcome by a method due to Evvard[10]. A feature of such mixed supersonic wings is that there is an area in the plane of the wing (see Fig.2), the flow at any point of which affects parts of the wing in its downstream Mach cone, while the flow at this point is in turn affected by parts of the wing in its upstream Mach cone. Evvard assumes that the wing can be regarded as being extended over this region by a membrane the slope of which is determined by the upwash at any point of it. This unknown upwash constitutes a new dependent variable. A distribution of sources over the wing and membrane is then postulated, determining the velocity potential at any point, and the velocity potential and unknown upwash are then made to satisfy two conditions, one being the known boundary condition derived from the motion of the wing, and the other the fact that the pressure is continuous at any point not on the wing and therefore in particular at all points on the membrane. This leads to two integral equations in two unknowns. These can be solved when certain assumptions, in effect that the unsteady motion of the wing has small accelerations, are made. In general there will be more than one of these disturbed areas, which must be regarded as being covered by a membrane, in the plane of the wing.

The/

The method is confined to cases in which these do not interact. The motion is not assumed to be simple-harmonic, but if this is assumed the method is valid to the first order in frequency. (See Moskowitz and Moeckel[15].) According to Stewart and Ting-Yi Li[51] the method is valid for steady and oscillating wings but not for general unsteady motions, but Miles[24] disagrees with this conclusion, apparently justifiably.

(vi) Moskowitz and Moeckel[15] have applied a modification of Evvard's technique to obtain a theory, valid to the first power of frequency, for wings with supersonic leading and trailing edges and streamwise tips. (See Fig. 3.)

(vii) General discussions and solutions for particular cases using methods similar to those mentioned above have been given by E. A. Krasilishchikova[13], and Haskind and Falkovitch[14], but no numerical results are given.

(viii) Germain and Bader[11] have given a method whereby a simple-harmonic solution of the oscillating equation can be derived immediately from a steady solution. The problem is then reduced to determining a steady solution with appropriate boundary conditions. It is however necessary that the solution can be expressed as the sum of a series of conical flows and then in general the problem reduces to the solution of an infinite number of steady flow problems. The same authors have also discussed the method in Ref. 12.

(ix) Gardner[9] has shown that the problem of a moving rectangular wing (not necessarily having simple-harmonic motion) may be made to depend on the solution of two steady problems of supersonic flow. It seems possible that this might be extended to other plan forms.

(x) Temple[18] and Stewartson[19] have applied the method of Laplace and Fourier transforms to various particular cases. This method seems to be capable of dealing with wings with supersonic trailing edges. Temple gives results for a pitching rectangular wing, a quasi-stationary delta wing with supersonic leading edges oscillating in pitch and vertically, and for a quasi-stationary delta wing with subsonic leading edges performing vertical oscillations, rolling, and pitching. Stewartson has also considered the last case including terms of higher order in the frequency.

§4. The Oscillating Rectangular Wing

The rectangular wing performing pitching oscillations has been considered by Watkins[16,17], Acum[8], Temple[18], Stewartson[19], Miles[24] and Stewart and Ting-Yi Li[51], and except for very low aspect ratio wings the flutter derivatives may be considered as known, subject to the limitations of the linearized theory, for frequency parameters and Mach numbers covering most of the values likely to be required for flutter problems. (See Table I.) The negative aerodynamic damping which occurs in the two-dimensional case still occurs but is reduced by finite aspect ratio.

Other motions of the wing, such as rolling and distortions of various kinds, appear not to have been considered in relation to flutter derivatives. However, it seems possible that some of the methods used above might be applicable to motions of this sort.

The stability derivatives are known. Harmon[20] has given formulae and graphs for these.

§5. The Oscillating Delta Wing

(i) Supersonic Leading Edge

As this is a purely supersonic problem it can be treated by methods (i) and (ii) of §3. Temple[18] has given a solution for the case of small frequency.

(ii) Subsonic Leading Edges

Temple[18] has given a solution for low frequency pitching, rolling, and vertical oscillations, and Robinson[7] has treated the pitching delta wing for general frequencies but gives no numerical results. No solution seems to be available for rolling other than for small frequencies.

(iii) Values of the derivatives (not yet published) have been computed by Mangler for the delta wing with subsonic or supersonic leading edges for the case of low frequency, and these show that for certain combinations of Mach number, axis position, and leading edge sweep, the damping of pitching oscillations may be negative.

(iv) The stability derivatives for a delta wing (purely supersonic or mixed supersonic) are known (Refs. 21, 22 and 29). Numerical values may be obtained from Refs. 21 and 29.

§6. Other Plan Forms

Any wing whose edges are all supersonic may be treated by the methods of Refs. 2 or 6. In this case the evaluation of the velocity potential on the wing reduces to the evaluation of an integral over part of the surface of the wing.

There appears to be no general method of dealing with wings whose edges are subsonic, in which case the methods of Refs. 2 and 6 lead to integral equations.

Sometimes the pressure distribution over a wing can be obtained by considering it as part of a wing for which a solution is known, provided that the flow over the parts omitted does not affect that over the remainder, thus the pressure distribution over a pointed tapered wing with supersonic trailing edges can be obtained from that over a delta wing. This is evidently of only limited application.

Moskowitz and Moeckel[15] have given a theory (first order in frequency) for wings of plan form of the type shown in Fig.3, i.e., supersonic leading and trailing edges and streamwise tips, pitching and performing vertical translational oscillations.

Miles[25] has considered the case of an infinite oscillating swept wing (supersonic leading edge) and also the case of an oscillating aileron on a rectangular wing[27].

§7. Gusts

All the foregoing remarks apply to wings performing simple-harmonic or at least continuous motions relative to the airstream, but it is also possible to consider the effect on the wing of a sudden change in its motion such as occurs when a sharp edged gust is encountered.

W. J. Strang/

W. J. Strang[30] has considered transient solutions of the linearized equation of supersonic flow by building them up from solutions corresponding to a source emitting a finite amount of fluid in an infinitesimal interval of time (pulse source) and to a source commencing to emit fluid at a given time. The solutions thus built up correspond to supersonic sources, doublets and vortices and combinations of sources and doublets. In Ref.31 the same author obtains results for the growth of lift following entry into gusts and change of incidence by building up such fundamental fields of flow.

The problem of transient supersonic flow has also been considered by J. W. Miles[32] who obtains the forces on a two-dimensional thin aerofoil entering a sharp-edged gust, having a sudden change of incidence, and a sudden flap deflection, by starting from the harmonic oscillation solution and integrating with respect to frequency. Miles gives curves showing the transient change of lift and moment in these cases. He concludes that (subject to the limitations of linearized theory), "It does not appear that the transient behaviour of a supersonic airplane due to sudden pitching or flap deflection will present serious problems unless dynamic stability is marginal......".

Heaslet and Lomax[42] have also dealt with the case of a two-dimensional aerofoil entering a sharp-edged gust or having a sudden change of angle of incidence.

Miles has also considered the infinite swept aerofoil with supersonic edges[33], obtained by postulating a distribution of sources over the plane of the wing. This leads to the result that the forces on a swept wing are equal to those of an unswept wing with equal (streamwise) chord and (streamwise) distribution of incidence, multiplied by the cosine of the angle of sweepback.

The case of a two-dimensional aerofoil which moves from one incidence to another in a finite time has been treated by Chang[34]. The method is that of source distribution over the surface of the wing.

The corresponding three-dimensional problems are not so well covered though Miles[35] has derived formulae for the average transient lift and moment acting on thin rectangular and delta wings entering a sharp-edged gust, from the corresponding simple-harmonic solutions by means of general formulae connecting such motions.

A typical example of the results obtained by the methods described above is shown in Fig.4 (taken from Heaslet and Lomax[42]). This shows the variation of the incrementary lift on a two-dimensional wing, free to move in the vertical direction without pitching, on entering a sharp-edged gust.

§8. Accelerated Motion

The motion of an aerofoil accelerating in the transonic range has been treated by Gardner and Ludloff[37,38] and an accelerating wedge by Biot[36], using linearized theory.

The supersonic case has been considered by Robinson[7].

§9. Experimental Work

Although a certain amount of experimental work has been done on transonic and supersonic flutter, it is difficult to reduce it to any systematic form. The principal methods used are:-

(i) Experiments in wind tunnels.

(ii) Experiments on wings attached to rockets.

(iii) Experiments on wings attached to falling missiles.

(iv) Experiments on models attached to aircraft in flight.

(v) Observations of performance of full-size aircraft in flight.

F. Smith[41] has discussed the various methods available, but further consideration of them is outside the scope of this paper.

The work of Bratt and Chinneck[39] in wind tunnels has been discussed in §2.

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TABLE I/

TABLE I

Leading-edge Derivative Coefficients for Complete Rectangular Wing

Aspect Ratio, $A \geqslant \frac{2}{\sqrt{M^2-1}}$

	M	$\delta\ell_z$	$\delta\ell_{\dot{z}}$	$\delta\ell_\alpha$	$\delta\ell_{\dot{\alpha}}$	δm_z	$\delta m_{\dot{z}}$	δm_α	$\delta m_{\dot{\alpha}}$
$\lambda = 0.2$									
	1.2	-0.156	-2.034	-2.074	+3.201	+0.116	+1.325	+1.357	-2.381
	1.4	-0.042	-1.006	-1.016	+0.705	+0.031	+0.666	+0.674	-0.528
	1.6	-0.019	-0.629	-0.634	+0.270	+0.014	+0.417	+0.421	-0.202
	1.8	-0.011	-0.441	-0.443	+0.131	+0.008	+0.293	+0.295	-0.098
$\lambda = 0.4$									
	1.2	-0.504	-1.445	-1.584	+2.568	+0.365	+0.862	+0.972	-1.858
	1.4	-0.156	-0.904	-0.944	+0.654	+0.115	+0.585	+0.617	-0.484
	1.6	-0.074	-0.593	-0.612	+0.258	+0.055	+0.389	+0.404	-0.192
	1.8	-0.043	-0.423	-0.434	+0.127	+0.032	+0.279	+0.288	-0.095
$\lambda = 0.6$									
	1.4	-0.311	-0.756	-0.840	+0.575	+0.227	+0.469	+0.535	-0.420
	1.6	-0.155	-0.538	-0.578	+0.239	+0.115	+0.345	+0.377	-0.176
	1.8	-0.093	-0.396	-0.420	+0.120	+0.069	+0.257	+0.276	-0.089
	2.0	-0.062	-0.304	-0.320	+0.068	+0.046	+0.199	+0.211	-0.051

Continued/

TABLE I (continued)

	M	ℓ_z	$\ell_{\dot z}$	ℓ_α	$\ell_{\dot\alpha}$	m_z	$m_{\dot z}$	m_α	$m_{\dot\alpha}$
$\lambda = 0.2$	1.2	+0.126	+2.803	+2.846	-1.696	-0.083	-1.349	-1.382	+1.102
	1.4	+0.041	+1.999	+2.013	-0.020	-0.027	-0.989	-0.999	+0.011
	1.6	+0.020	+1.585	+1.591	+0.293	-0.013	-0.788	-0.793	-0.196
	1.8	+0.012	+1.328	+1.332	+0.372	-0.008	-0.662	-0.665	-0.248
$\lambda = 0.4$	1.2	+0.390	+2.292	+2.442	-1.127	-0.241	-0.979	-1.089	+0.653
	1.4	+0.150	+1.880	+1.933	+0.043	-0.097	-0.901	-0.940	-0.039
	1.6	+0.076	+1.537	+1.563	+0.310	-0.050	-0.753	-0.772	-0.210
	1.8	+0.045	+1.303	+1.318	+0.379	-0.030	-0.643	-0.655	-0.254
$\lambda = 0.6$	1.4	+0.287	+1.712	+1.820	+0.137	-0.179	-0.779	-0.858	-0.114
	1.6	+0.154	+1.465	+1.520	+0.337	-0.099	-0.699	-0.740	-0.231
	1.8	+0.094	+1.264	+1.297	+0.390	-0.061	-0.615	-0.639	-0.262
	2.0	+0.062	+1.111	+1.133	+0.395	-0.040	-0.545	-0.561	-0.264

Explanation

$$\text{Lift} = L = \rho V^2 c^2 e^{ipt} \left[z_0' \left\{ A(\ell_z + i\lambda\, \ell_{\dot z}) + (\delta\ell_z + i\lambda\, \delta\ell_{\dot z}) \right\} + \alpha' \left\{ A(\ell_\alpha + i\lambda\, \ell_{\dot\alpha}) + (\delta\ell_\alpha + i\lambda\, \delta\ell_{\dot\alpha}) \right\} \right]$$

$$\text{Moment} = \mathcal{M} = \rho V^2 c^3 e^{ipt} \left[z_0' \left\{ A(m_z + i\lambda\, m_{\dot z}) + (\delta m_z + i\lambda\, \delta m_{\dot z}) \right\} + \alpha' \left\{ A(m_\alpha + i\lambda\, m_{\dot\alpha}) + (\delta m_\alpha + i\lambda\, \delta m_{\dot\alpha}) \right\} \right]$$

$\alpha = \alpha' e^{ipt}$ = incidence, $cz_0 = cz_0' e^{ipt}$ = depression of leading edge,

$\lambda = \dfrac{pc}{V}$, c = chord.

$\mathcal{M} > 0$ implies that $\mathcal{M}$ tends to raise the leading edge and depress the trailing edge.

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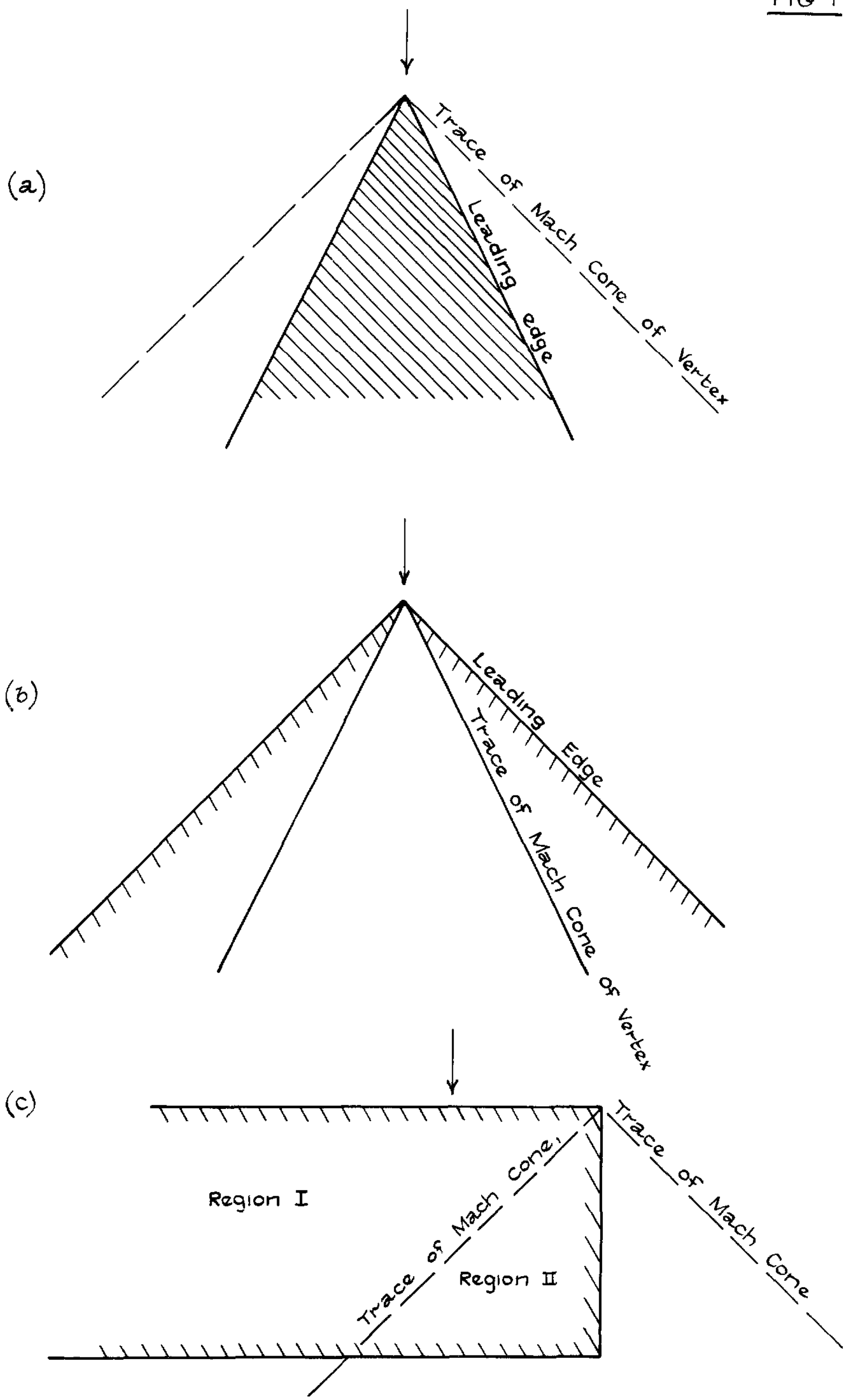
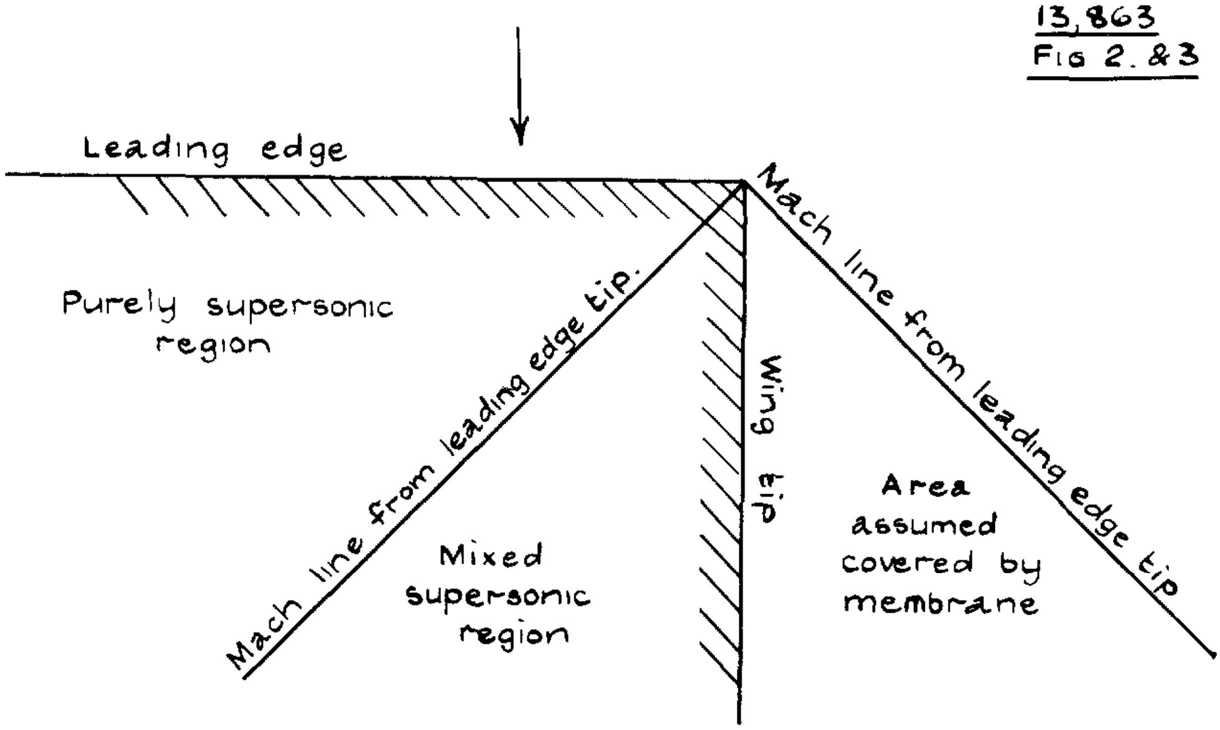
FIG 1

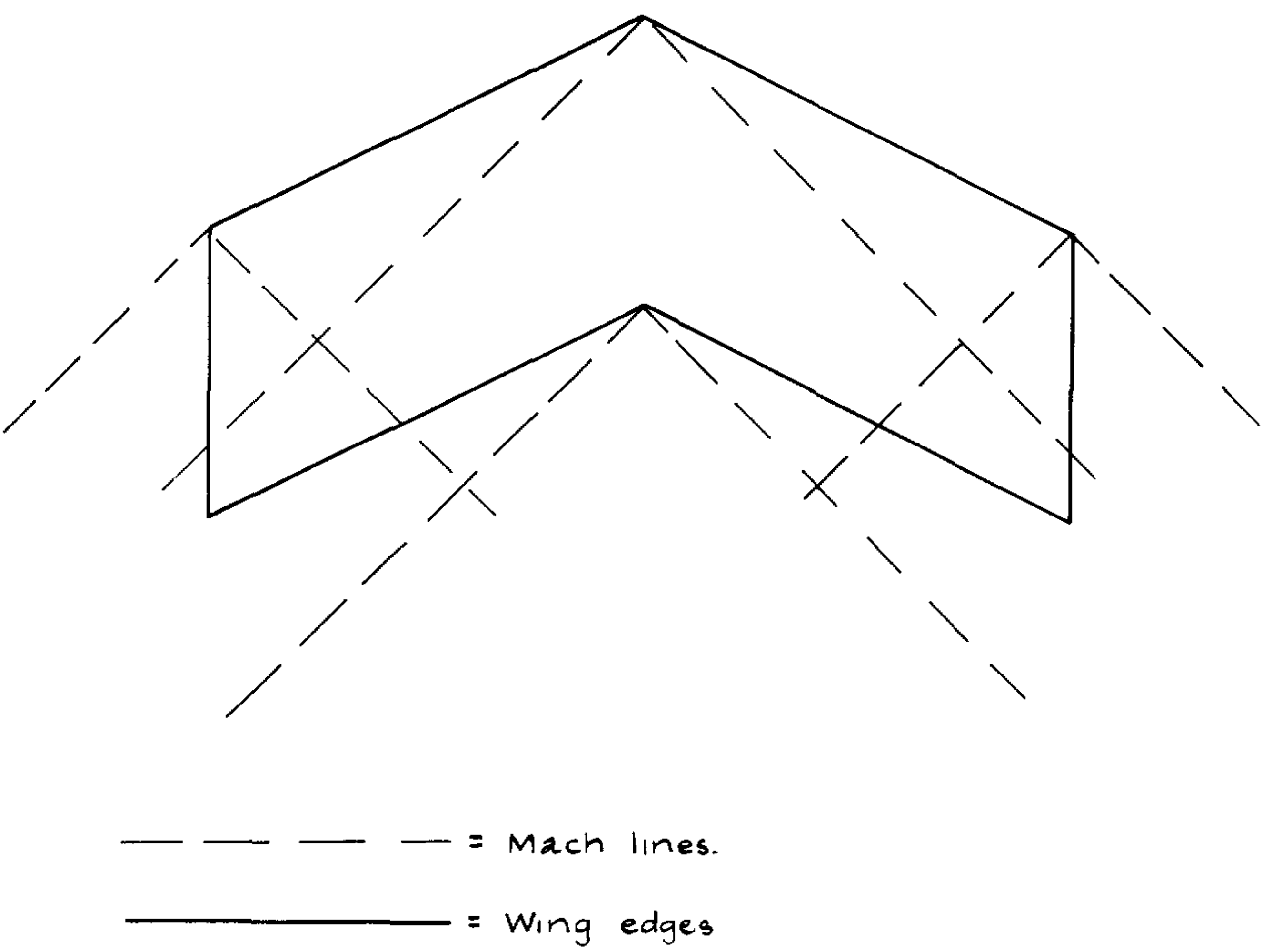


Types of Supersonic Flow Regions

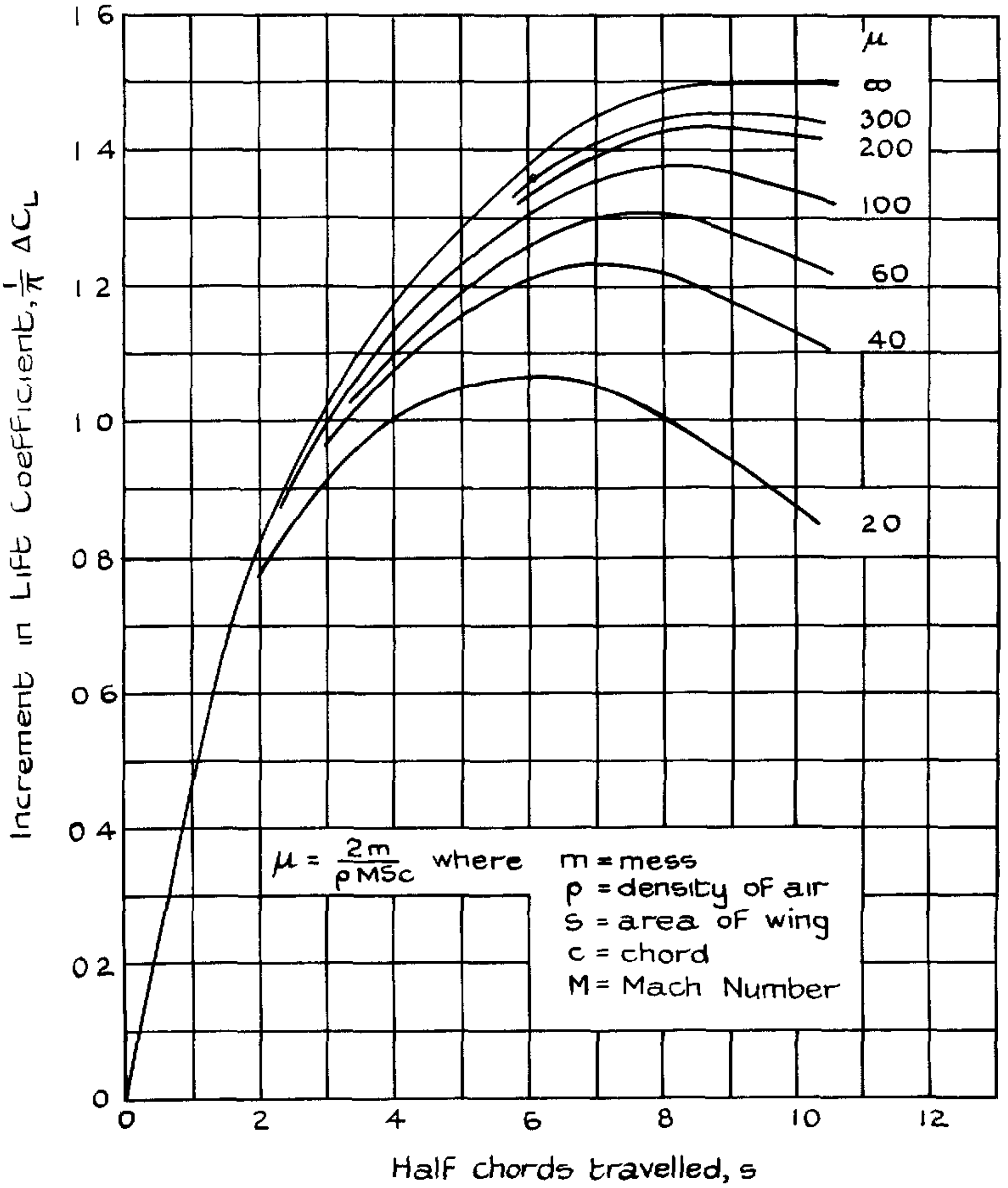
E.P.S



Evvard's method (§3(v))



Wing of the type considered by Moskowitz and Moeckel (Ref. 15) See §6



Variation of increment of Lift Coefficient during flight through a sharp edged unit gust. (Unrestrained wing) M = 1 31.

C.P. No. 85
(13,863)
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